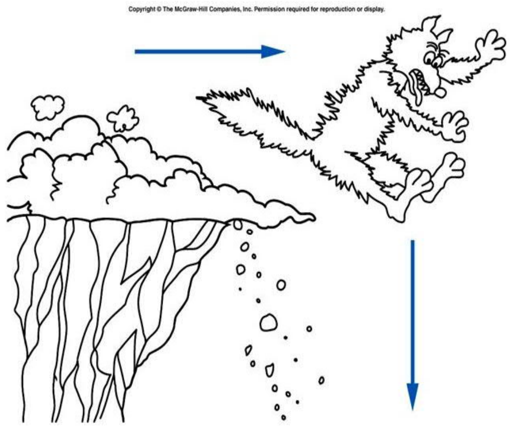
Projectile Motion:

A Cartoon Coyote Falling off a Cliff



We need to consider the horizontal and vertical motions separately projectile motions. Time is the same for both motions.

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1.1 Projectile Motion

ABSTRACT

This study notes have been developed and written to meet the scope and syllabus of all the content of the SACE Stage 2 Physics 2020. The goal of this topic is to enable students not just to recognize concepts, but to work with them in ways that will be useful in final exam.

Muralikumar, M.E., CPEngg., RPEQ.,

Stage 2 Study Notes

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Subtopic 1.1 Projectile Motion

Students are introduced to the theories and quantitative methods used to describe, determine, and explain projectile motion, both in the absence of air resistance and in media with resistive forces.

Students study projectile motion, through a range of investigations to understand how the principles are applied in the contexts of sports, vehicle designs, and terminal speed.

Science Understanding	Possible Contexts
Uniformly accelerated motion is described in terms of relationships between measurable scalar and vector quantities, including displacement, speed, velocity, and acceleration. Motion under constant acceleration can be described quantitatively using the following formulae: • $v = u + at$ • $s = ut + \frac{1}{2}at^2$ • $v^2 = u^2 + 2as$. Projectile motion can be analyzed quantitatively by treating the horizontal and vertical components of the motion independently. • Construct, identify, and label displacement, velocity, and acceleration vectors. • Use vector addition and subtraction to calculate net vector quantities. • Resolve velocity into vertical and horizontal components, using $v_H = v\cos\theta$ and $v_v = v\sin\theta$ for the horizontal and vertical components respectively. • Determine the velocity at any point, using trigonometric calculations or a scale diagram.	Explanation of the difference between scalar and vector quantities and methods of measurement of these quantities is covered in Stage 1, Topic 1: Linear Motion and Forces. This uses the concept of acceleration developed in Stage 1, Subtopic 1.1: Motion under Constant Acceleration. Use trigonometric calculations and scale diagrams to determine quantities, using vector addition and subtraction. Given a diagram showing the path of a projectile, draw vectors to show the forces acting on the projectile, as well as the acceleration and velocity vectors. Use a projectile launcher to investigate the effect of launch angle or launch height on range.
An object experiences a constant gravitational force near the surface of the Earth, which causes it to undergo uniform acceleration. • Explain that the acceleration of a projectile is always downwards and independent of its mass. • Explain that, in the absence of air resistance, the horizontal component of the velocity is constant.	Demonstrate the independence of acceleration due to gravity on mass using NASA footage of dropping a hammer and feather on the Moon: https://www.youtube.com/watch?v=5C5_dOEyAfk

<table>
<tr><th>Science Understanding</th><th>Possible Contexts</th></tr>
<tr><td>The motion formulae are used to calculate measurable quantities for objects undergoing projectile motion.
• Calculate the time of flight when a projectile is launched horizontally.
• Calculate the time of flight and the maximum height for a projectile when the launch height is the same as the landing height.
• Calculate the horizontal range of a projectile.
• Explain qualitatively that the maximum range occurs at a launch angle of 45° for projectiles that land at the same height from which they were launched.
• Describe the relationship between launch angles that result in the same range.
• Describe and explain the effect of launch height, speed, and angle on the time of flight and the maximum range of a projectile.
• Analyze multi-image representations of projectile paths.</td><td>Investigate the ‘monkey and the hunter’ problem both quantitatively and qualitatively.
Use video footage to analyses projectile motion in a variety of contexts.
Analyze the constant horizontal component of the velocity of the projectile qualitatively and quantitatively, using various recording technologies.
Model and demonstrate that the maximum range occurs at a launch angle other than 45° when the launch height is different to the landing height.
In terms of projectile motion, analyses footage of students undertaking a sport like shot put.
Use concepts from projectile motion to analyses sporting activities such as aerial skiing, golf, javelin, shot put, and various ball sports.</td></tr>
<tr><td rowspan="2">When a body moves through a medium such as air, the body experiences a drag force that opposes the motion of the body.
• Explain the effects of speed, cross-sectional area, and density of the medium on the drag force on a moving body.
• Explain that terminal velocity occurs when the magnitude of the drag force results in zero net force on the moving body.
• Describe situations such as skydiving and the maximum speed of racing cars where terminal velocity is achieved.
• Describe and explain the effects of air resistance on the vertical and horizontal components of the velocity, maximum height, and range of a projectile.
• Describe and explain the effects of air resistance on the time for a projectile to reach the maximum height or to fall from the maximum height.</td><td>Determine the terminal velocity of a spherical object by dropping it into a viscous liquid.
Determine the drag coefficients by dropping coffee filters or cupcake holders. By manipulating the mass and recording the time taken to reach the ground, use the air resistance formula to calculate the drag coefficient.
Discuss the conclusions of experiments comparing swimming in syrup with swimming in water:
http://www.nature.com/news/2004/040920/full/news040920-2.html</td></tr>
<tr><td>Explore examples of the way that scientists have been able to develop solutions affecting aerodynamics (such as shape, texture, and spin) of different objects like balls, planes, and cars.</td></tr>
</table>

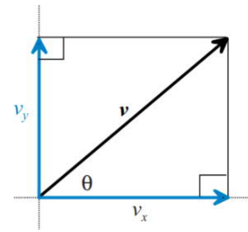
1 Components of Vectors

1.1 Resolving a Vector into its Components

All vectors can be resolved into two mutually perpendicular components.

Most commonly, we find the components of vectors in the horizontal and vertical directions. But this does not have to be so.

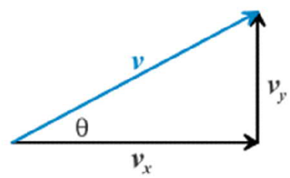
The components of the vector can be found in any pair of directions – the only limiting condition is that the two directions must be perpendicular to each other.



Hence a vector is equal to the sum of its components

1.2 Finding a Vector from its Components

If the components of a two-dimensional vector are known, then the Magnitude of the vector can be found by application of Pythagoras' theorem.



$$\boldsymbol{v}^2 = {v_x}^2 + {v_y}^2$$

$$\boxed{\boldsymbol{v} = \sqrt{{v_x}^2 + {v_y}^2}}$$

The Direction of the vector can be found by the application of right-angled triangle rule.

$$tan\,\theta = \frac{v_y}{v_x}$$

$$\boxed{\theta = tan^{-1}\left(\frac{v_y}{v_x}\right)}$$

Note

- Motions in two dimensions can be analysed as the vector sum of two mutually perpendicular, one-dimensional motions.

We use the components of force, acceleration, velocity and displacement in these directions.

The components can be taken in any two, mutually perpendicular, directions

- No vector can have any effect, action or component in a direction perpendicular to itself.

Because of this we can treat the horizontal and vertical components of the parameters of the motion as completely independent of each other. Thus, we can treat two-dimensional motions as two independent motions in the horizontal and vertical directions. That is, two separate, independent, one-dimensional motions.

- All vectors can be resolved into their horizontal and vertical components.
- Given its components, the original vector can be found.

1.3 Motion Parameters

In Motion study, the following vector parameters are used.

Displacement ‘S’

The displacement ‘S’ of a body is equal to the vector sum of its Easterly and Northerly components.

$$\boxed{\boldsymbol{S} = ut + \frac{1}{2}at^2}$$

$$\boxed{\boldsymbol{S} = vt - \frac{1}{2}at^2}$$

$$\boxed{\boldsymbol{S} = \frac{1}{2}(v - u)t}$$

Velocity ‘v’

The velocity ‘v’ of a body is equal to the vector sum of its Easterly and Northerly components

$$\boxed{\boldsymbol{v} = u + at}$$

$$\boxed{\boldsymbol{v^2} = u^2 + 2aS}$$

Acceleration ‘a’

Acceleration is constant and always directed vertically ↓ downwards.

Force ‘F’

The Force ‘F’ of a body is equal to the vector sum of its Easterly and Northerly components

$$\boxed{\boldsymbol{F} = ma}$$

2 Projectile Motion

A projectile is any object that has been projected (or launched or thrown or fired) at some angle into the air, near the surface of the earth. The subsequent motion of this object is a parabola, by neglecting the air resistance.

The analysis of this two-dimensional projectile motion relies on the following techniques.

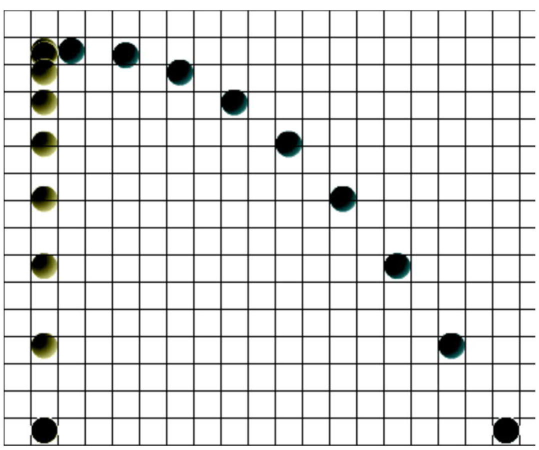
- Resolve the motion into two perpendicular one-dimensional motions.
- Treating these two components of the motion independent of each other.

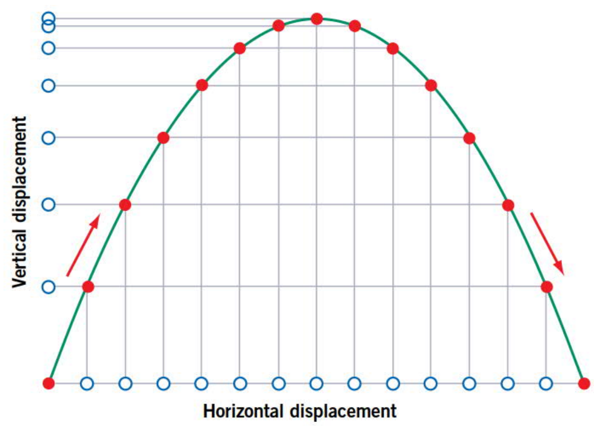
2.1 Vertical and Horizontal Components of Velocity

A projectile's velocity is a vector and can be resolved into two components (Horizontal v_x and Vertical v_y). These components are vectors at right angles to each other.

A multi-image photograph can be used to demonstrate the behaviour of the components.

In a multi-image photograph, the time difference between images is constant.





Since the projectile moves the same horizontal distance every time, its horizontal component of velocity must be constant. $s_x = u_x\, t$.

Therefore, to draw the images of the projectile with equal horizontal spacing between them.

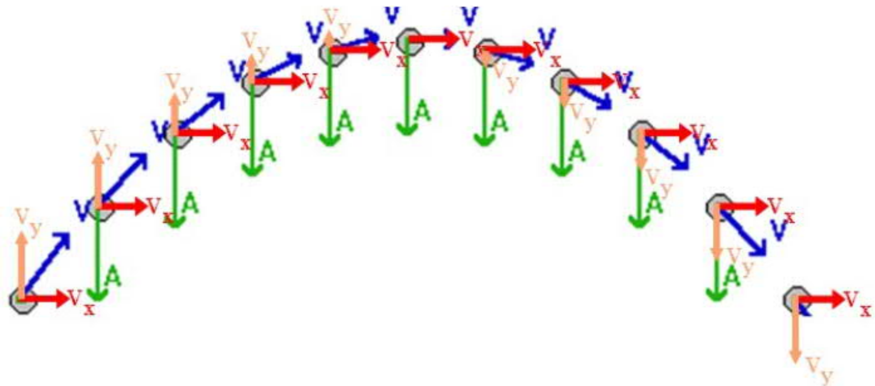
Both objects have zero initial vertical component of velocity to begin with. They fall to the ground in the same amount of time, which shows:

- Horizontal motion and vertical motion are independent of each other
- Projectiles have the same acceleration as a vertically free-falling object

Note

- The path of the projectile is parabola.
- A projectile is an object moving through the air in a curved trajectory with no propulsion system.
- The horizontal component of the velocity remains constant throughout the flight.
- The vertical component of the velocity decreases as the projectile rises; and increases as the projectile falls:
- At the topmost point the vertical component of the velocity is zero.
- The direction of motion of velocity is always at a tangent to the parabola at all times. The magnitude and direction of velocity changes with time.
- Acceleration is constant and always directed vertically ↓ downwards.
- The force of air resistance always acts opposite to the direction of motion of velocity.
- A projectile is an object that only moves under the influence of gravity
- The vertical and horizontal components of motion are independent
- Projectile motion is motion with a constant horizontal velocity combined with a constant vertical acceleration.
- The horizontal and vertical motions of a projectile are independent of each other except they have a common time.
- Projectile motion problems can be solved by applying the constant velocity equation for the horizontal component of the motion and the constant acceleration equations for the vertical component of the motion.
- Pythagoras' theorem can be used to determine the actual speed of the projectile at any point.

3 Velocity



- The velocity vector, v, is always at a tangent to the parabola.
- The direction of the velocity and the magnitude of the velocity changes with time.
- The horizontal component of the velocity v_x remains constant throughout the motion. Therefore, when sketching, the horizontal components are all in the same length.
- The vertical component of the velocity v_y changes with time, decreasing in magnitude while the projectile is rising.
- Once the projectile passes the highest point of its flight, the vertical component reverses direction, and increases in magnitude in the downward direction.
- At the highest point of its flight the velocity of the projectile is in the horizontal direction. The vertical component of the velocity here, is zero ($v_y = 0$). The actual velocity at this point is equal to horizontal component velocity only. ($v = v_x$).
- The vertical and horizontal components $v_x\ and v_y$ of velocity at any point are known, the actual velocity v at that point can be found by vector addition of these components, i.e. $\boldsymbol{v} = v_x + v_y$
- The acceleration is constant and always directed vertically downward.

3.1 Resolution of Initial Velocity into Components

In case of horizontal projection, the initial vertical component of velocity ($u_y = 0$) is zero and the horizontal component is equal to the initial velocity. i.e. ($u_x = u$).

In all other cases, where the body is initially projected with initial velocity u at angle θ to the horizontal, the following components applicable. In which the horizontal component of velocity remains constant throughout the motion.

- Horizontal component of initial velocity ($u_x = u\cos\theta$)
- Vertical component of initial velocity ($u_y = u\sin\theta$)

3.2 Determination of Final Velocity

The final velocity of the body can be found by the vector addition of the horizontal and vertical components of final velocity. Note that the horizontal component of final velocity is nothing but the horizontal component of the initial velocity.

$(v_x = u_x)$

The vertical component of final velocity can be found by using Newton's first law of motion.

$(v_y = u_y + a_y t)$.

By using Pythagoras theorem, the final velocity v^2 is

$(v^2 = {v_x}^2 + {v_y}^2)$

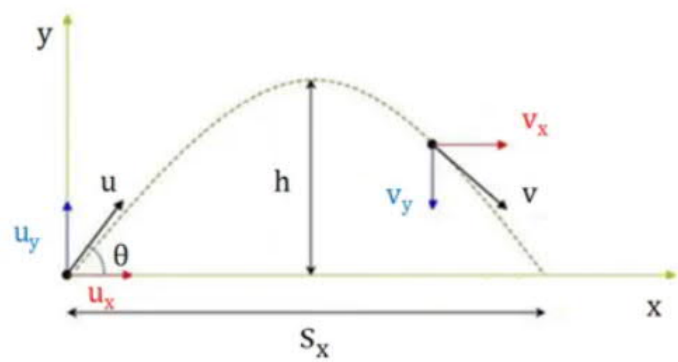
The angle will be by trigonometry

$$\boxed{\tan\theta = \frac{v_y}{v_x}}$$

$$\boxed{\theta = \tan^{-1}\frac{v_y}{v_x}}$$

Note:

The velocity of a projectile body at any time is at a tangent to the parabolic path traced out. The vertical and horizontal components of velocity at any time, when we add them vectorially, will give the instantaneous velocity at that point.



$u_x = u \cos\theta$

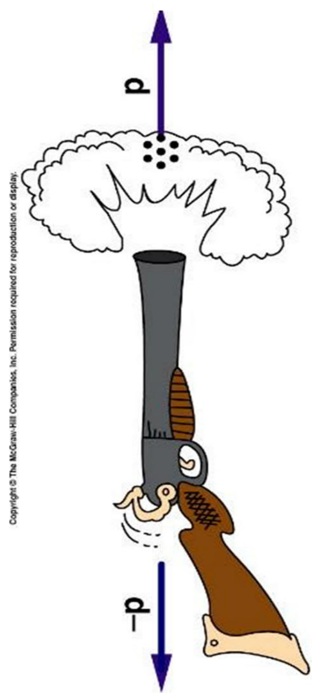
$u_y = u \sin\theta$

$u_x = v_x$ (horizontal component)

$v_y = 0$ (at the max height 'h')

$S_x = u_x * t$ (as normal equation)

$S_y = h$ (where $v_y = 0$ and $v_x = u_x$)



PHYSICS

1.2 Forces & Momentum

ABSTRACT

This study notes have been developed and written to meet the scope and syllabus of all the content of the Stage 2 Physics 2020. The goal of this topic is to enable students not just to recognize concepts, but to work with them in ways that will be useful in final exam.

Muralikumar ME., CPEngg., RPEQ.,

Stage 2 Study Notes

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Subtopic 1.2: Forces and Momentum

Students learn to use force and acceleration vectors to discuss Newton's Laws of Motion and are introduced to the vector nature of momentum. They explain the law of the conservation of momentum in terms of Newton's Laws and develop skills in vector addition and subtraction within this context.

Science Understanding	Possible Contexts
Momentum is a property of moving objects; it is conserved in an isolated system and may be transferred from one object to another when a force acts over a time interval. Newton's Second Law of Motion can be expressed as two formulae, $\vec{F} = m\vec{a}$ and $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$, where $\vec{p} = m\vec{v}$ is the momentum of the object. • Derive $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ by substituting the defining formula for acceleration $\left(\vec{a} = \frac{\Delta\vec{v}}{\Delta t}\right)$ into Newton's Second Law of Motion, $\vec{F} = m\vec{a}$, for particles of fixed mass. (The net force, $\vec{F}$, and hence the acceleration, $\vec{a}$, are assumed to be constant. Otherwise, average or instantaneous quantities apply.) • Draw vector diagrams in which the initial momentum is subtracted from the final momentum, giving the change in momentum, $\Delta\vec{p}$. • Solve problems (in both one dimension and two dimensions) using the formulae $\vec{F} = m\vec{a}$, $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$, and $\vec{p} = m\vec{v}$. Newton's Third Law of Motion, $\vec{F}_1 = -\vec{F}_2$, in conjunction with the Second Law expressed in terms of momentum, implies that the total momentum of a system of two interacting particles, subject only to the force of each one on the other, is conserved.	Many of these ideas have been introduced in Stage 1 through one-dimensional situations. The focus here should be on two-dimensional situations. Consider the development of the discovery of neutrinos. This uses the concepts of acceleration and force developed in Stage 1, Subtopics 1.1: Motion under Constant Acceleration and 1.2: Forces and Momentum. Use the conservation of momentum to determine the speed of a projectile by firing it into a trolley. Investigate how the law of conservation of momentum was used to predict the existence of neutrinos. Explore perspectives in the public debate about the economics of space exploration. Is government funding likely to be maintained? Research the most appropriate types of spacecraft propulsion for journeys to different destinations, considering technical challenges and speculative technologies.

Science Understanding	Possible Contexts
• Derive a formula expressing the conservation of momentum for two interacting particles by substituting $\vec{F}_1 = \frac{\Delta \vec{p}_1}{\Delta t}$ and $\vec{F}_2 = \frac{\Delta \vec{p}_2}{\Delta t}$ into $\vec{F}_1 = -\vec{F}_2$. • Use the law of the conservation of momentum to solve problems in one and two dimensions. • Analyze multi-image representations to solve conservation of momentum problems, using only situations in which the mass of one object is an integral multiple of the mass of the other object(s). The scale of the representations and the flash rate can be ignored. The conservation of momentum can be used to explain the propulsion of spacecraft, ion thrusters, and solar sails. • Use the conservation of momentum to describe and explain the change in momentum and acceleration of spacecraft due to the emission of gas particles or ionized particles. • Use the conservation of momentum to describe and explain how the reflection of particles of light (photons) can be used to accelerate a solar sail. • Use vector diagrams to compare the acceleration of a spacecraft, using a solar sail where photons are reflected with the acceleration of a spacecraft, using a solar sail where photons are absorbed.	

1 Newton's Law

1.1 Newton's first law of motion

First law states that "A body at rest remains rest, and a moving body continues to move with constant velocity unless an external force acted on it".

It is concluded that forces cause changes to the state of motion of a body. The force acting on a body determines the change in the velocity of the body (either in magnitude or direction or both) and hence the acceleration is given by

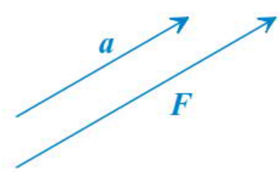
$$\boldsymbol{a} = \frac{\triangle \boldsymbol{v}}{\triangle \boldsymbol{t}}$$

1.2 Vector form of Newton's second law of motion

Second law states that "The acceleration of a body is directly proportional to the force acting on it".

Therefore, the force acting on a body is directly proportional to the acceleration that it produces. $ie \quad \boldsymbol{F} \propto \boldsymbol{a} \quad hence \quad \boldsymbol{F} = \boldsymbol{ma}$. The constant of proportionality is the inertia (mass) of the body.

The acceleration 'a' of an object is always in the same direction as the force 'F' acting on it. Thus, it can be expressed the relationship between the magnitudes and the directions of force and acceleration by the vector relation: $\vec{\boldsymbol{F}} = \boldsymbol{m}\vec{\boldsymbol{a}}$



The implications of this vector equation are that the direction of the vector $\vec{\boldsymbol{F}}$ is the same as the direction of the vector $\vec{\boldsymbol{a}}$. The magnitude of the vector $\vec{\boldsymbol{F}}$ is equal to 'm' times the magnitude of the vector $\vec{\boldsymbol{a}}$.

2 Newton's Second Law of motion in terms of Momentum

2.1 The Concept of Momentum

Newton's second law of motion states that 'The acceleration of a body is directly proportional to the force acting on it". $ie \quad \boldsymbol{F} \propto \boldsymbol{a} \quad hence \quad \boldsymbol{F} = \boldsymbol{ma}$. The constant of proportionality is the inertia (mass) of the body.

However, the acceleration of a body is defined as the rate of change of its velocity.

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v - u}{t} = \frac{v_f - v_i}{t}$$

By substituting in (Newton's second law) formula,

$$F = m\,\frac{v_f - v_i}{t} = \frac{mv_f - mv_i}{t}$$

$$F = \frac{\Delta mv}{t}$$

In the above derivation, the assumption was that the net force F and the acceleration a are constant.

Thus, the force on a body is a measure of the rate at which the quantity (Δmv) changes. This new property of the body is called momentum, usually given the symbol $'p'$

Thus momentum $'p'$ = mass x velocity i.e. $\boldsymbol{p} = \boldsymbol{mv}$

$$Therefore \quad \boldsymbol{F} = \frac{\Delta \boldsymbol{p}}{\boldsymbol{t}} = \frac{p_f - p_i}{t} \quad where\ [p = mv]$$

Momentum is a vector quantity having both magnitude and directions. ($\boldsymbol{kg.m.s^{-1}}$)

2.2 Newton's second law of motion in terms of Momentum

There are two different ways in which Newton's second law of motion is commonly stated.

Statement 1: - Force is given by the product of the acceleration and the mass of the body.

$$\boldsymbol{F} = \boldsymbol{ma}$$

Statement 2: - Force is given by the rate of change of the momentum of the body.

$$\boldsymbol{F} = \frac{\Delta \boldsymbol{p}}{\Delta \boldsymbol{t}}$$

Hence to calculate the force acting on a body, one needs to calculate the change of momentum by vector means, divide by the time over which the force acts in order to cause this momentum change.

Alternative unit for Momentum

From the second statement, Newton's law can be rearranged as $\Delta \boldsymbol{p} = \boldsymbol{F}\Delta \boldsymbol{t}$. it can be seen that momentum 'p' is the product of time and force. (unit → sN)

2.3 Problem Solving Method for 2D Collisions with a Surface

There are two approaches to problems involving 2D collisions with a surface.

Approach 1

The first approach involves finding the change in velocity ($\Delta v = v - u = v_f - v_i$). Then we use this to find the acceleration ($a = \Delta v/\Delta t$) and then, using the first expression of Newton's second law ($F = ma$), we find the force acting on the collision body. The direction of acceleration and the force are the same as the direction of the vector Δv

Approach 2

The second approach involves finding the change in momentum ($\Delta p = p_f - p_i$). Then using the second expression of Newton's second law ($F = \Delta p/\Delta t$), we can find the force acting on the collision body. From this using first expression ($F = ma$), we can find the acceleration of the body during the collision. The direction of acceleration and the force are the same as the direction of the vector Δp

Note

- By Newtons third law of motion, the direction of the force on the surface is always equal and opposite to the direction of the force on the colliding body
- If a body collides with a surface and rebounds with the same speed and at the same angle, then $\Delta \boldsymbol{v}$, $\Delta \boldsymbol{p}$, $\boldsymbol{a}$ $\boldsymbol{and}$ $\boldsymbol{F}$ are all at the right angles to the surface and away from it.

2.4 Particles rebounding from a surface

When a particle collides with a surface, it changes direction, i.e. it accelerates. As the particle hits the surface it exerts a force on the surface. From Newton's third law, the surface exerts an equal and opposite force on the particle, and it is this force which causes the particle to change direction. Since a force is acting there must also be a change in momentum.

Collision at right angles to a surface, rebounding with no change in speed

A particle of mass m collides at right angles with a surface. The particle rebounds at right angles at the same speed, i.e. $v_f = -v_i$

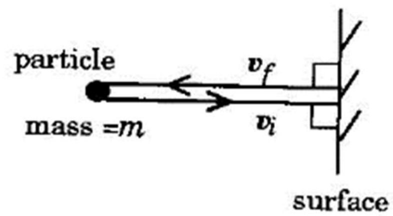
$v = v_f - v_i$

$\Delta v = v_f + v_f$

$\Delta v = 2v_f$

$Therefore \quad a = \frac{\Delta v}{\Delta t}$

$a = \frac{2v_f}{\Delta t}$



The average acceleration of the particle is in the direction of v_f, which is at right angles away from the surface.

From Newton's second law the force of the wall on the particle is given by

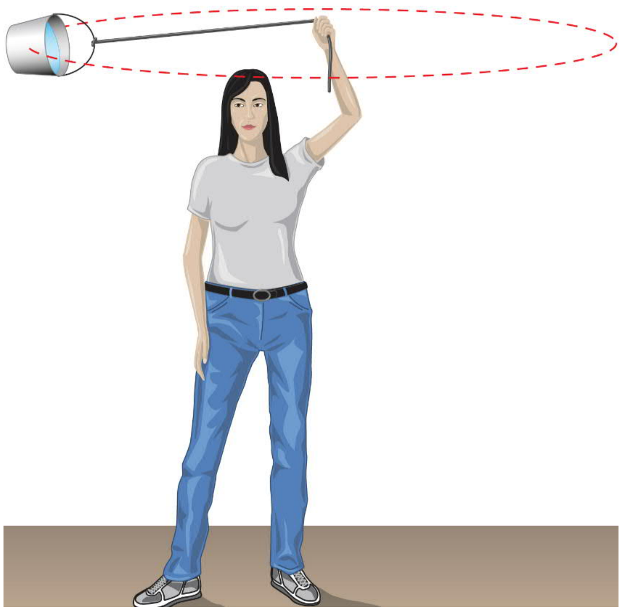
$$F = ma = \frac{\Delta p}{\Delta t}$$

$$F = m \times \left(\frac{2v_f}{\Delta t}\right)$$

$$F = \frac{2mv_f}{\Delta t}$$

$$F = \frac{2p_f}{\Delta t}$$

Therefore, the force, the acceleration and the change in momentum are all in the same direction, i.e. at right angles away from the wall.



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1.3 Circular Motion

ABSTRACT

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Subtopic 1.3: Circular motion and Gravitation

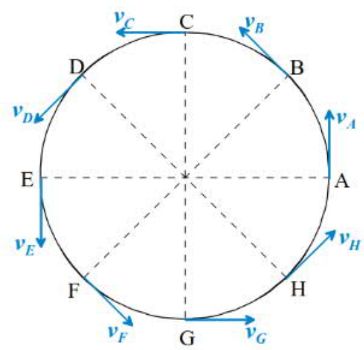
Students investigate the circular motion that results from centripetal acceleration in a variety of contexts, including satellites and banked curves. Students are introduced to the concepts of Newton's Law of Universal Gravitation and Kepler's Laws of Planetary Motion.

They explore extra-terrestrial phenomena that can be explained using Newton's Law of Universal Gravitation and Kepler's Laws of Planetary Motion.

An object moving in a circular path at a constant speed undergoes uniform circular motion. This object undergoes centripetal acceleration, which is directed towards the center of the circle. The magnitude of the centripetal acceleration is constant for a given speed and radius and given by $a = \frac{v^2}{r}$. The formula $v = \frac{2\pi r}{T}$ relates the speed, v, to the period, T, for a fixed radius. • Solve problems involving the use of the formulae $a = \frac{v^2}{r}$, $v = \frac{2\pi r}{T}$, and $\vec{F} = m\vec{a}$. • Use vector subtraction to show that the change in the velocity, $\Delta\vec{v}$, and hence the acceleration, of an object over a very small-time interval is directed towards the center of the circular path.	This uses the concepts of acceleration and force developed in Stage 1, Subtopics 1.1: Motion under constant acceleration and 1.2: Forces. Describe situations in which the centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force. Investigate the force causing centripetal acceleration, using a tube, stopper, washers, and connecting string. Find and test the speed required for a marble to 'loop the loop', using flexible railing or a slot-car set.
On a flat curve, the friction force between the tires and the road causes the centripetal acceleration. To improve safety, some roads are banked at an angle above the horizontal. • Draw a diagram showing the force vectors (and their components) for a vehicle travelling around a banked curve. • Explain how a banked curve reduces the reliance on friction to provide centripetal acceleration.	Explore the benefits and limitations in the design and use of banked curves, such as in velodromes, motor racing circuits, amusement park rides, and high-speed train tracks.

1 Uniform Circular Motion

When a particle moves on a circular path with a constant speed, then its motion is known as uniform circular motion. The magnitude of the velocity in a circular motion is constant but the direction changes continuously.



If a body moves with constant speed 'v', in a circle of fixed radius 'r', its velocity 'v' at any point has magnitude 'v' and is directed at a tangent to the circle at that point.

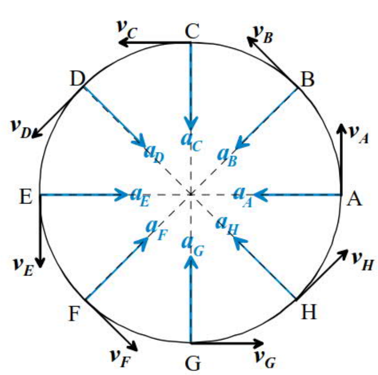
Note

- The speed of the body is constant; the velocity of the body is continually changing (in Direction).
- The different velocity vectors all have the same length (Magnitude)
- The velocity at any point is perpendicular to the radius at that point.

2 Centripetal Acceleration

In all uniform circular motion, there is an acceleration directed radially inwards to the centre of the circle. This is called centripetal acceleration.

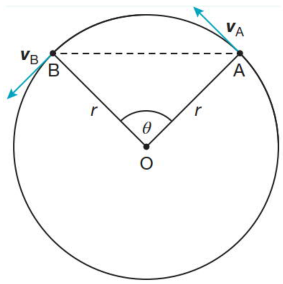
$$\boxed{a_c = \frac{v^2}{r}}$$

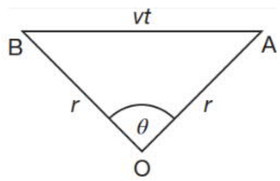


2.1 The Magnitude of Centripetal Acceleration

Using vector geometry, it is possible to derive an expression for the magnitude of an object's centripetal acceleration.

Consider a body rotating in a circle of radius r with constant speed v. In moving from point, A to point B, the object moves through an angle θ as shown.

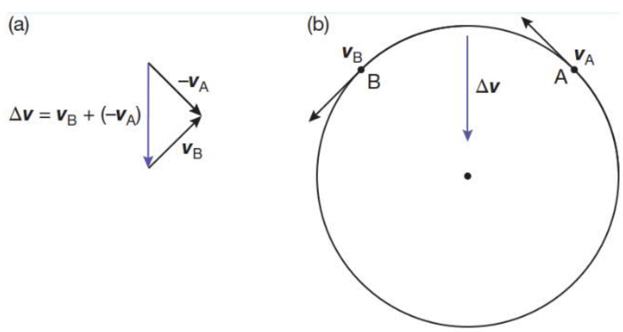




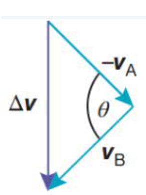
1

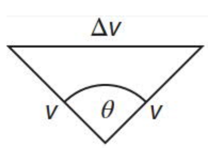
OAB is seen to form an isosceles triangle with OA = OB. The third side is formed by a line, or chord, joining A with B. When the angle θ is small, the length of this chord is virtually the same as the length of the arc which also joins these two points. As this distance is covered by an object travelling with an average speed v in a time t, the length of AB can be calculated using s = v t.

Turning our attention to the vector addition of $-v_A$ and v_B as shown, we find that the angle made by the two vectors is equal to θ, the angular separation of A and B.



As the object is moving with uniform circular motion, the lengths of the vectors v_B and $-v_A$ are identical (both equal to the object's average speed v) and so form two sides of an isosceles triangle as shown





2

As the triangles shown in both figures 1 and 2 are isosceles triangles with the same angle θ between their equal sides, we can describe them as being similar triangles — they can be thought of as the same triangle drawn on two different scales.

As the triangles are similar, the ratio of their sides must be constant, so:

$$\frac{\Delta v}{vt} = \frac{v}{r}$$

$$\frac{\Delta v}{\mathrm{t}} = \frac{v^2}{r}$$

AS $a = \frac{\Delta v}{t}$

$$\boxed{\boldsymbol{a_c} = \frac{\boldsymbol{v^2}}{\boldsymbol{r}}} \quad (Acceleration)$$

Thus, we find that the magnitude of the centripetal acceleration ac of an object moving with uniform circular motion

Note

- This formula only gives the magnitude of the centripetal acceleration. Its direction is along the radius, directed towards the centre of the circle.
- Acceleration is always at right angles to the tangent of the circle.
- The centripetal acceleration depends upon the tangential speed of body and the radius at which it is travelling. Thus, for a given speed the radius of acceleration $\boldsymbol{a_c}$ is (constant) fixed.
- Acceleration only changes the direction of velocity but not in magnitude.

2.2 The Direction of Centripetal Acceleration

Consider a body of mass 'm' rotating a circle with constant speed 'v'. As the speed of the body is constant, its kinetic energy does not change. Therefore, no work is done on the body by the force acting on it.

But work done is given by $W = F\,s\cos\theta$.

Where F is force applied, s is distance travelled and θ is the angle between the force and the direction of motion of body.

Therefore as $W = F\,s\cos\theta = 0$, then $\cos\theta$ must be equal to zero, and so θ must be 90^0. Thus, the force must be at right angles to the direction of the motion of the body.

Thus, the acceleration must be directed in towards the centre of the circle.